

Anisotropies in the Cosmic Neutrino Background after WMAP 5-year Data

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We search for the presence of cosmological neutrino background (CNB) anisotropies in recent WMAP 5-year data using their signature imprinted on modifications to cosmic microwave background (CMB) anisotropy power spectrum. By parametrizing the neutrino background anisotropies with the speed viscosity parameter c_{vis} , we find that the WMAP 5-year data alone provide only a weak indication for CNB anisotropies with $c_{\text{vis}}^2 > 0.06$ at the 95% confidence level. When we combine CMB anisotropy data with measurements of galaxy clustering, SN-Ia Hubble diagram, and other cosmological information, the detection increases to $c_{\text{vis}}^2 > 0.16$ at the same 95% confidence level. Future data from Planck, combined with a weak lensing survey such as the one expected with DUNE from space, will be able to measure the CNB anisotropy parameter at about 10% accuracy. We discuss the degeneracy between neutrino background anisotropies and other cosmological parameters such as the number of effective neutrinos species and the dark energy equation of state.

I. INTRODUCTION

The recent results on the cosmic microwave background (CMB) anisotropies from the five-year data of the WMAP mission have once again confirmed the basic predictions of the standard cosmological model [1]. In addition to an improved determination of several key cosmological parameters such as the scalar spectral index and the optical depth to reionization with a better control of systematics, the new WMAP 5-year data have provided, probably for the first time, a clear indication for the presence of anisotropies in the cosmic neutrino background (CNB) at more than 95% confidence level.

The CNB is a clear prediction of the standard cosmological model, though a direct detection of the cosmic neutrinos remain extremely challenging. These neutrinos decouples from the primeval plasma at temperature $T \sim 1$ MeV before electron-positron annihilation. A relic neutrino background is expected today at a temperature of $T_\nu = (4/11)^{1/3} T_\gamma$ (where $T_\gamma = 2.728 K$ is the CMB blackbody temperature) with an energy density of $\rho_\nu \sim 0.58 N_{\text{eff}} T_\nu^4$. In the standard cosmological model $N_{\text{eff}} = 3.045$ and any hint for $N_{\text{eff}} > 0$ can be therefore considered as an indication for the CNB.

Recent combined analyses of CMB data with other cosmological observables have already provided striking evidence for the presence of this neutrino background. A combination of WMAP three-year data with measurements of Hubble expansion rate, for example, provided the constraint that $N_{\text{eff}} = 3.7 \pm 1.1$ at 95% confidence level [2], with other analyses finding consistent results (see e.g. [3]). The recent WMAP 5-year data analysis has reported an estimate of N_{eff} , but only including CMB data in the analysis [1]. Their results is then the most conservative, though it is now clear that neutrinos are certainly an important energy component of our universe.

In this paper we reanalyze, in light of the new WMAP 5-year data release and their results, some

further properties of the neutrino background, namely the presence of anisotropies in the background and not just the effective number of neutrino species. Although inflationary anisotropies in the CNB at the level of $\sim 10^{-5}$ are expected in the standard scenario, a direct detection of this anisotropy is significantly more challenging than a simple detection of the relic CNB today. The expected anisotropies in the CNB, however, affect the CMB anisotropy angular power spectrum at level of $\sim 20\%$ through the gravitational feedback of the neutrino free-streaming damping and anisotropic stress contributions [4]. This allows an indirect detection of the neutrino background anisotropy through its signature in the CMB.

A way to parameterize the anisotropies in the CNB has been introduced in [5] with the “viscosity parameter” c_{vis}^2 , which controls the relationship between velocity/metric shear and anisotropic stress in the CNB. In the standard scenario $c_{\text{vis}}^2 = 1/3$, anisotropies are present in the CNB and approximate the radiative viscosity of neutrinos. The case $c_{\text{vis}}^2 = 0$, on the contrary, cuts the Boltzmann hierarchy of CNB perturbations at the quadrupole, forcing a perfect fluid solution with no CNB anisotropies but only density and velocity (pressure) perturbations. Observationally determining $c_{\text{vis}}^2 > 0$ would therefore provide a strong indication for the existence of CNB anisotropies, as argued in [6]. This possibility is studied in several papers that made use of early CMB data [7] with varying levels of indirect detection of the CNB anisotropy.

In this paper we reanalyze the constraints on neutrino anisotropies in light of the new WMAP 5-year data. We combine CMB data with existing large-scale structure cosmological information to get an overall estimate on the viscosity parameter and to constraint it better, for the first time, at a confidence level greater than 95%. Moreover, we also study how future experiments as the Planck satellite CMB mission, possibly combined by future weak lensing data, will be able to constrain this parameter. Our paper is organized as follows: in the next Section, we dis-

cuss the CMB and large-scale structure data analysis and present our results based on existing data in Section III. We also forecast the expected errors in Section IV and conclude with a summary of our results in Section V.

II. ANALYSIS

The method we adopt for this analysis is based on the publicly available Markov Chain Monte Carlo package `cosmomc` [8] with a convergence diagnostics based on the Gelman and Rubin statistic. We sample the following eight-dimensional set of cosmological parameters, adopting flat priors on them: the physical baryon and Cold Dark Matter, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling, θ_s , the scalar spectral index n_s , the overall normalization of the spectrum A at $k = 0.05 \text{ Mpc}^{-1}$, the optical depth to reionization, τ , the number of massless neutrinos N_{eff} and, finally, the viscosity parameter c_{vis} introduced above with the prior $c_{\text{vis}}^2 \leq 1/3$. Simultaneously we also use a cosmic age top-hat prior as $10 \text{ Gyr} \leq t_0 \leq 20 \text{ Gyr}$.

Furthermore, we consider purely adiabatic initial conditions, we impose flatness and we treat the dark energy component as a cosmological constant. We include the five-year WMAP data [1] (temperature and polarization) with the routine for computing the likelihood supplied by the WMAP team. Together with the WMAP data we also consider the small-scale CMB measurements of ACBAR [9], CBI [10], and BOOMERANG-2K [11]. In addition to the CMB data, we include the Supernovae Legacy Survey data [12], the real-space power spectrum of galaxies from the Sloan galaxy redshift survey (SDSS) [13], and 2dF galaxy power spectrum [14], and, in a separate analysis, constraints from the real-space power spectrum of red galaxies from the Sloan galaxy redshift survey (SDSSlrg) [15].

Our analysis considers 2 different scenarios: firstly, we test for anisotropies in a relativistic background comprising $N_{\text{eff}} = 3$ massless neutrino species. Secondly, we allow the number of massless neutrinos to vary. In both cases, we allow for variations in the viscosity parameter c_{vis} .

We consider here 3 separate datasets: the first with WMAP 5-year only (WMAP5), the second with all existing CMB data (ALL) combined with data from Sloan red galaxy power spectrum (ALL+SDSSlrg), and the third with all CMB information combined with the SDSS power spectrum (ALL+SDSS). In the last two cases we have also imposed the Gaussian Hubble Space Telescope prior[16]: $h = 0.72 \pm 0.08$ and also a weak big-bang nucleosynthesis prior[17]: $\omega_b h^2 = 0.022 \pm 0.002$ (1σ). In the computation of the CMB spectra we have allowed for the lensing modification [18]. As above for the other parameters we

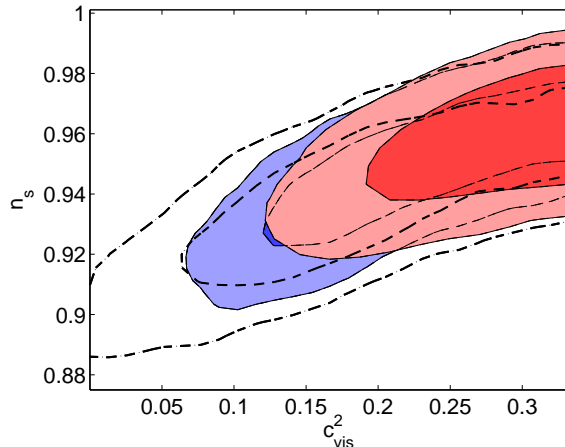


FIG. 1: Joint 2-dimensional posterior probability contour plots in the $c_{\text{vis}}^2 - n_s$ plane, showing the 68% and 95% confidence level contours from the WMAP 5-year data alone (empty contours), ALL+SDSSlrg dataset (blue contours) and ALL+SDSS dataset (red contour). Including the full Sloan dataset is necessary to improve the limits placed on c_{vis}

have adopted flat priors.

III. RESULTS

A. Standard background of $N_{\text{eff}} = 3$ massless neutrinos.

In Figure 1 we plot constraints obtained from our analysis in the $c_{\text{vis}}^2 - n_s$ plane in the case of $N_{\text{eff}} = 3$ massless neutrinos. The WMAP 5-year data alone is able to put only a weak constrain on c_{vis} , due to the degeneracy with the spectral index highlighted in [19], $c_{\text{vis}}^2 > 0.06$ at 95% confidence level. Including the full cosmological dataset and the red galaxy spectrum (ALL+SDSSlrg) the degeneracy is partially broken enough to obtain $c_{\text{vis}}^2 > 0.11$ (95% confidence level). Including the full Sloan + 2dF dataset (ALL+SDSS) we obtain a stronger constrain with $c_{\text{vis}}^2 > 0.16$ at the same 95% confidence level. The difference in log-likelihood between the best fit for the standard case and the case with no anisotropies ($c_{\text{vis}}^2 = 0$) is $\Delta\chi^2 = 15.2$. Therefore we can conclude that the case where the NB does not have anisotropies above the first moment is quite clearly disfavored.

B. General background of massless neutrinos.

We now relax the assumption of a standard background of relativistic neutrinos by treating N_{eff} as a free parameter and thus check the ability of the data to put simultaneous constraints both on the

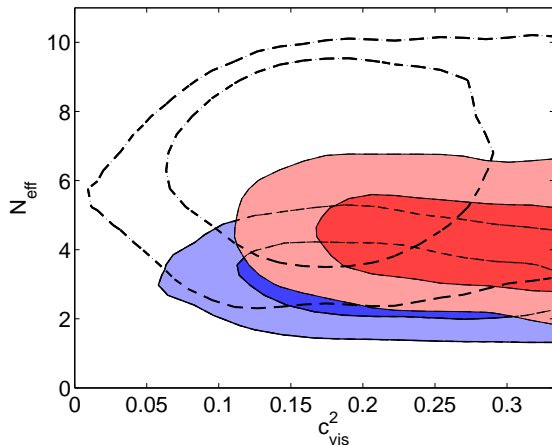


FIG. 2: Joint 2-dimensional posterior probability contour plots in the $c_{\text{vis}}^2 - N_{\text{eff}}$ plane, showing the 68% and 95% contours from the WMAP5 data alone (empty contours), ALL+SDSSlrg dataset (blue contours) and ALL+SDSS dataset (red contour).

background number of neutrinos and on the presence of anisotropies in it. In Figure 2 we show the 2-dimensional constraints in the plane $c_{\text{vis}}^2 - N_{\text{eff}}$.

Dataset	c_{vis}^2	$N_{\text{eff}} = 3$	N_{eff}	c_{vis}^2
WMAP5	≥ 0.06		$6.4^{+2.8}_{-3.2}$	≥ 0.07
ALL+SDSSlrg	≥ 0.11		$3.2^{+1.7}_{-1.5}$	≥ 0.11
ALL+SDSS	≥ 0.16		$4.4^{+2.1}_{-1.8}$	≥ 0.15

TABLE I: The 95% confidence level limits on N_{eff} and lower limit at 95% c.l. on c_{vis}^2 for WMAP5 data alone, ALL+SDSSlrg and ALL+SDSS (see text for details).

As we can see, the WMAP data alone can provide a weak indication for a background of relativistic neutrinos, giving $3.2 < N_{\nu} < 9.2$ at 95% c.l., and it is able to put also a weak constraint on c_{vis} , giving $c_{\text{vis}}^2 > 0.07$ at 95% c.l.. Considering the ALL+SDSSlrg dataset we obtain $1.7 < N_{\nu} < 4.9$ at 95% c.l. and $c_{\text{vis}}^2 > 0.11$ at 95% c.l.. Including the full Sloan dataset increases the constraining on c_{vis}^2 :

$$c_{\text{vis}}^2 > 0.15 \quad (95\% \text{ c.l., 1-tail}) \quad (1)$$

$$2.6 < N_{\nu} < 6.5 \quad (95\% \text{ c.l., 2-tails}). \quad (2)$$

The difference in log-likelihood between the best fit for the standard case and the case with no anisotropies ($c_{\text{vis}}^2 = 0$) is $\Delta\chi^2 = 19.4$.

IV. FORECAST

In this section we investigate how future experiment could improve the constraints on the viscosity parameter c_{vis}^2 , the number of relativistic species N_{eff} and the

scalar spectral index n_s together with other cosmological parameters. Several future experiments will focus on the observation of weak gravitational lensing with large-area galaxy surveys. While weak lensing surveys can improve our understanding of neutrino physics [20], weak lensing observations help in this analysis by providing a way to break certain degeneracies by improving the determination of parameters such as n_s . CMB experiments, however, are more sensitive to c_{vis}^2 , though in CMB alone, this parameter is degenerate with other quantities. It is therefore interesting to study how the combination of a planned CMB experiment, mainly Planck, with weak lensing surveys can improve the constraints on the CNB anisotropy parameters.

A. Fisher matrix

To explore this we perform a Fisher matrix analysis [21] where the Fisher matrix is

$$F_{\alpha\beta} \equiv \left\langle -\frac{\partial^2 \ln L}{\partial p_{\alpha} \partial p_{\beta}} \right\rangle, \quad (3)$$

where L is the likelihood function of the set of parameters p_i . Here we adopt a cosmological model with ten parameters. We combine the Fisher matrix of Planck with the Fisher matrix of three different weak lensing surveys to study the dependence of the constraints on the survey parameters since weak lensing surveys are still under development. According to the Cramer-Rao inequality for unbiased estimators this is the best statistical error that one can obtain for the generic parameter p_{α} :

$$\sigma_{p_{\alpha}} \geq \sqrt{(F^{-1})_{\alpha\alpha}}. \quad (4)$$

For a CMB experiment the Fisher matrix is given by:

$$F_{\alpha\beta}^{CMB} = \sum_{l=2}^{l_{\text{max}}} \sum_{i,j} \frac{\partial C_l^i}{\partial p_{\alpha}} (\text{Cov}_l)_{ij}^{-1} \frac{\partial C_l^j}{\partial p_{\beta}}, \quad (5)$$

where the $C_l^{\alpha\beta}$ are the well known power spectra for the temperature (TT), temperature-polarization (TE), E-mode polarization (EE), and B-mode polarization (i and j run over TT, EE, TE, BB) and Cov_l is the spectra covariance matrix. We use the experimental configuration of the Planck satellite to compute the Fisher matrix for CMB anisotropies. The specifications for this experiment are listed in Table II. We use $l_{\text{max}} = 1500$ to calculate the sum in (5).

The future weak lensing surveys allow the possibility to "tomographically" reconstruct the distribution of foreground matter responsible for distortions in the images of distant background galaxies [22]. One can therefore divide the survey into various redshift bins and reconstruct the convergence weak lensing power

	Chan.	FWHM	$\Delta T/T$	$\Delta P/T$
$f_{sky} = 0.65$	100	9.5'	2.5	4.0
	143	7.1'	2.2	4.2
	217	5.0'	4.8	9.8

TABLE II: Experimental specifications for the Planck satellite. Channel frequency is given in GHz, FWHM in arcminutes, and noise in 10^{-6} .

spectrum $P_{ij}(\ell)$ at multipole ℓ ; the subscripts i and j are referred to the redshift bins. The convergence power spectrum contains the non linear matter power spectrum P_{nl} at redshift z , obtained correcting the linear power spectrum $P(k, z)$.

With these assumptions the Fisher matrix for weak lensing is given by [23]:

$$F_{\alpha\beta} = f_{sky} \sum_{\ell} \frac{(2\ell+1)\Delta\ell}{2} \frac{\partial P_{ij}}{\partial p_{\alpha}} C_{jk}^{-1} \frac{\partial P_{km}}{\partial p_{\beta}} C_{mi}^{-1}, \quad (6)$$

where $\Delta\ell$ is the step used for ℓ and:

$$C_{jk} = P_{jk} + \delta_{jk} \langle \gamma_{int}^2 \rangle n_j^{-1}, \quad (7)$$

is the lensing covariance. Here, we ignore non-Gaussian contribution to the covariance [?] as we are interested in obtaining an estimate on the ability of these experiments to measure CNB parameters.

In the last expression γ_{int} is the rms intrinsic shear (and we assume $\langle \gamma_{int}^2 \rangle^{1/2} = 0.22$ consistent with high fidelity space-based imaging data and future ground-based data) and n_j is the number of galaxies per steradian belonging to j th bin:

$$n_j = 3600d \left(\frac{180}{\pi} \right)^2 \hat{f}_j, \quad (8)$$

where d is the number of galaxies per square arcminute and \hat{f}_j is the fraction of sources belonging to the j th bin. In our analysis we use ℓ in the range $10 \leq \ell \leq 2000$, so that we are well within the linear to mildly non-linear regime of fluctuations and not to bias our results with uncertainties in predictions related to non-linear regime at $\ell > 2000$. We considered three different future weak lensing surveys, PanSTARRS [25], DES [26] and DUNE [27] experiments, with different sky coverage (f_{sky}), median redshift z_0 and number of sources per square arcminute d . We assume a radial distribution function of galaxies given by $D(z) = z^2 \exp[-(z/z_0)^{1.5}]$ where z_0 depends on the survey considered. The experimental characteristics of these survey are listed in table III.

Here, we account for uncertainties in the measured photometric redshifts of the galaxies, according to the approach of [28]: if $p(z_{ph}|z)$ is the probability that a galaxy with (real) redshift z is observed at photometric redshift z_{ph} then the distribution of galaxies in the i th bin is given by $D_i(z) = \int_{z_{ph,i}} dz_{ph} D(z) p(z_{ph}|z)$.

We choose the probability $p(z_{ph}|z)$ to be Gaussian: $p(z_{ph}|z) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{(z_{ph}-z)^2}{2\sigma_z^2}\right]$ with σ_z depending on the survey.

We then combine the Fisher matrix of Planck with the Fisher matrix of different weak lensing surveys to show how these surveys can improve the constraints coming from CMB. Since CMB anisotropies and weak lensing of distant galaxies have origin in two distant epochs of the history of the Universe we can consider them to be independent. Then, the total Fisher matrix is simply the sum of the Fisher matrix of CMB and that of weak lensing:

$$F_{\alpha\beta}^{TOT} = F_{\alpha\beta}^{CMB} + F_{\alpha\beta}^{WL}. \quad (9)$$

experiment	f_{sky}	z_0	d	σ_z
DES	0.13	0.8	10	0.05(1+z)
PanSTARRS	0.75	0.75	5	0.06(1+z)
DUNE	0.5	0.9	35	0.03(1+z)

TABLE III: Experimental specifications for the three weak lensing surveys of our analysis.

B. Results

To compute the convergence power spectrum of weak lensing we use the code CAMB with the option HALOFIT [29] to obtain the non linear matter power spectrum under the halo model [30]. Then we calculate P_{ij} and its derivatives with respect to cosmological parameters to construct the Fisher matrix. We consider the redshift range $0 \leq z \leq 3$ and divide it in 5 redshift bins of equal size.

For the CMB Fisher matrix we use a modified version of CAMB, to allow variation in c_{vis}^2 . The derivatives that appear in (5) and (6) are calculated from the target model. We use a set of nine cosmological parameters whose target values are: $\Omega_b h^2 = 0.0223$, $\Omega_c h^2 = 0.114$, $\Omega_{\Lambda} = 0.7$, $n_s = 1.0$, $\tau = 0.084$, $w = -1$, $A_s = 2.4 \cdot 10^{-9}$, $N_{eff} = 3.04$ and $c_{vis}^2 = 1/3$. We assume a flat universe imposing the flatness condition:

$$h = \sqrt{(\Omega_c h^2 + \Omega_b h^2)/(1 - \Omega_{\Lambda})}, \quad (10)$$

In table IV we report the 1σ uncertainties for the parameters of our model from Fisher analysis. These results shows that Planck alone will achieve better constraints on the cosmological parameters compared to weak lensing surveys, except for the dark energy parameters Ω_{Λ} and w which are very sensitive to the tomographic information coming from weak lensing. When combining CMB experiments with weak lensing surveys, the improvement in 1σ uncertainties is of more than one order of magnitude on Ω_{Λ} and w and of a factor 1.5-2 on the others parameters. This

holds even for the viscosity parameter c_{vis}^2 (although this parameter has no effect on the matter power spectrum and hence on the convergence power spectrum of weak lensing) because of the degeneracies between the parameters of the model, such as the degeneracies $n_s - c_{\text{vis}}^2$ and $N_{\text{eff}} - c_{\text{vis}}^2$.

The combination of CMB experiments with weak lensing surveys can put strong constraints on w , though CMB alone can constrain w only weakly. Moreover Figure 5 shows that there is not any relevant degeneracy between w and c_{vis}^2 and a measure of dark energy equation of state implies nothing for possible anisotropies in the neutrino background.

	Planck	DUNE	PanSTARRS	DES
$\sigma(\Omega_b h^2)$	0.00044	0.0059	0.018	0.026
$\sigma(\Omega_c h^2)$	0.0052	0.020	0.057	0.086
$\sigma(\Omega_\Lambda)$	0.12	0.0051	0.011	0.018
$\sigma(n_s)$	0.012	0.024	0.071	0.10
$\sigma(\tau)$	0.0061	--	--	--
$\sigma(w)$	0.39	0.036	0.096	0.15
$\sigma(\ln A_s)$	0.030	0.088	0.28	0.39
$\sigma(N_{\text{eff}})$	0.38	0.83	2.7	3.8
$\sigma(c_{\text{vis}}^2)$	0.075	--	--	--
Planck+				
	DUNE	PanSTARRS	DES	
$\sigma(\Omega_b h^2)$	0.00020	0.00022	0.00025	
$\sigma(\Omega_c h^2)$	0.0032	0.0036	0.0039	
$\sigma(\Omega_\Lambda)$	0.0027	0.0073	0.011	
$\sigma(n_s)$	0.0037	0.0051	0.0069	
$\sigma(\tau)$	0.0054	0.0057	0.0058	
$\sigma(w)$	0.015	0.031	0.043	
$\sigma(\ln A_s)$	0.012	0.029	0.016	
$\sigma(N_{\text{eff}})$	0.18	0.21	0.24	
$\sigma(c_{\text{vis}}^2)$	0.043	0.046	0.047	

TABLE IV: 1σ errors on the cosmological parameters of our model from Planck and weak lensing surveys alone (top) and from Planck combined with each survey (bottom).

In Figure 3, Figure 4 and Figure 5 we show the joint constraints on c_{vis}^2 with n_s , N_{eff} and w , from Planck alone and from Planck combined with the three weak lensing survey.

To obtain the confidence contours for any couple of parameters, we take the elements of the inverse Fisher matrix $(F^{-1})_{\alpha\beta}$ that corresponds to those parameters. This gives the correlation matrix of that couple of parameters and the eigenvalues and eigenvectors of this correlation matrix give the orientation and the size of the ellipsoid of confidence. This procedure is equivalent to marginalize on the remaining parameters (see also [31]).

In performing the Fisher analysis we are assuming that the likelihood is Gaussian and that the maximum of the likelihood is in the target model. The likelihood for the parameter c_{vis}^2 could not be exactly Gaussian, at least for $c_{\text{vis}}^2 > 1/3$. However expanding the likelihood in Taylor series around the target model we can see that the likelihood function is approximately Gaussian for $c_{\text{vis}}^2 \leq 1/3$, (i.e. it is dominated from quadratic terms), even if it could be not Gaussian globally. In fact usually L drops rapidly so that we can neglect the non-Gaussianity of the likelihood [32] and the constraints from Fisher analysis can be considered a good approximation.

The most considerable result of this analysis is that it is possible to detect neutrino background anisotropies even from Planck alone with $0.33 > c_{\text{vis}}^2 > 0$. The combination of Planck experiment with weak lensing surveys can decrease the statistical uncertainty on c_{vis}^2 about of a 35% (for Planck+Dune), while we find the combinations Planck+DES and Planck+PanSTARRS give similar constraints.

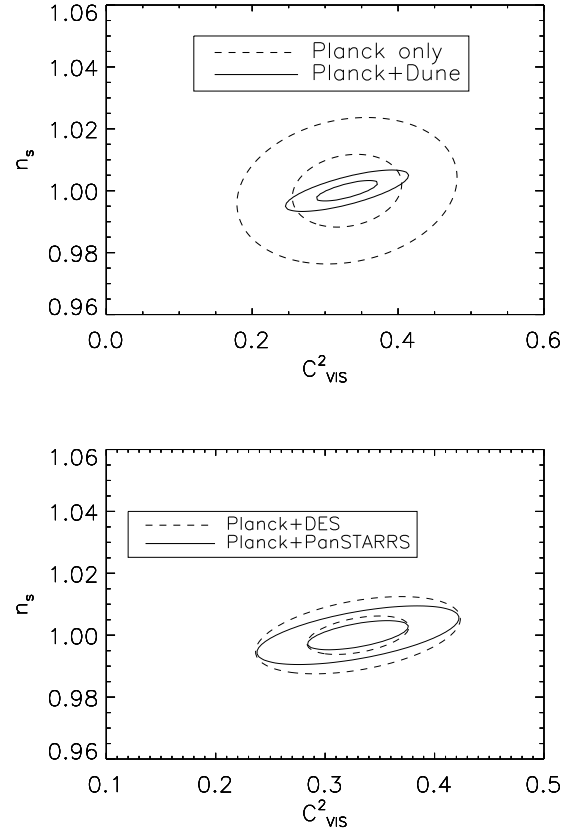


FIG. 3: Constraints for c_{vis}^2 and n_s from Planck alone and from Planck combined with DUNE experiment (top) and from Planck combined with others redshift surveys (bottom).

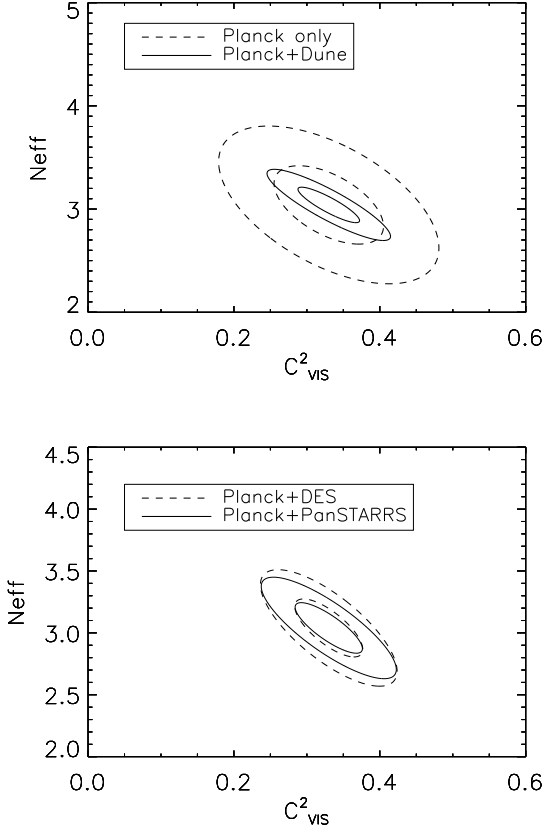


FIG. 4: Same as Figure 3 but for c_{vis}^2 and N_{eff}

V. CONCLUSIONS

In this paper we have re-analyzed the status of neutrino anisotropies in light of the new WMAP five-year data. We found that while the WMAP 5-year data alone are unable to provide significant evidence for those anisotropies, combination of CMB data with current large-scale structure cosmological data yields a detection at more than 2σ confidence level. Future cosmological data are certainly needed to confirm this

result.

In this respect we have performed a forecast for future CMB and weak lensing missions. We have found that a combination of DUNE and Planck will be able to yield a measurement of the c_{vis} parameter at $\sim 10\%$ level. Moreover, we studied the possible degeneracies with other cosmological parameters. Including uncertainties on c_{vis} will double the error bars on N_{eff} while the constraints on the equation of state w will remain practically unaffected. While the standard model predicts $c_{\text{vis}}^2 = 1/3$ several cosmological scenarios, from interacting neutrinos to early dark energy, can be considered that could bring a deviation in the measured value. A better detection of anisotropies in the neutrino background could therefore provide a useful test for those models and possibly indicate the pres-

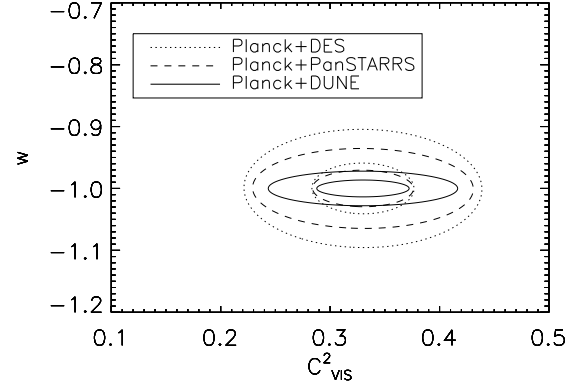


FIG. 5: Constraints on c_{vis}^2 and w from Planck combined with the three weak lensing surveys.

ence of new physics.

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